[12.8] Let  be independent 1‑forms in ℝ*n*. Let  be a simple *p*-form and be a simple *q*‑form. Show that 

**Proof**: Juergen Beckmann’s proof uses the antisymmetrized-coefficients expression for the wedge product and is well presented and very insightful. This problem can also be solved using the un-antisymmetrized-coefficients expression.

Let M = {1, 2, … , *n*}, **P**r… u be the set of permutations of the *p*-tuple (*r*, … , u) and **P** j … m the set of permutations of the *q*-tuple (*j*, … , *m*). As Juergen reminds us in his proof, the un-antisymmetrized expressions for *ϕ* and *χ* are



The un-antisymmetrized expression for *ϕ* ∧ *χ* is

 (3)

The un-antisymmetrized expression for  is

 (4)

Both expressions (3) and (4) have *n p+q* terms, and all terms are identical. Therefore

, which finishes the proof. 

**Note:** Juergen’s proof has several nice features and he clarifies issues only hinted at by Penrose.

First, he introduces an elegant notation to represent the antisymmetrization of two antisymmetrizations:

. (5)

I, for one, had a hard time figuring out where to put the outside brackets in order to express the outer antisymmetrization.

He also clarified the meaning of the RHS of (5). I considered expanding the RHS using a 2! permutation because I thought that the outer antisymmetrization was operating on the 2 quantities  and , but that was incorrect. Juergen’s explanation uses a permutation expression involving (*p* + *q*)! and I believe it translates to



Next he expresses the inner antisymmetrizations in terms of permutations:





Juergen then finished his proof with some clever re-writing of the permutations to eliminate the two inner antisymmetrizations (i.e., the 2nd and 3rd summation signs), leading to the (desired) expression for . My only purpose here was to attempt to translate his notation without his reference to a generic antisymmetric operator.

We provide two examples, one illustrating the permutation notation and the other illustrating the [12.8] problem statement.

**Example: Permutation Notation**

Let  be 1-forms in ℝ2. Then *n* = *p* = 2, M = {1, 2 }, and So **P** 12 = {1, 2} where



So, for example,

 

**Example: **

Let . Set  where . So .

Next, let  where 

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